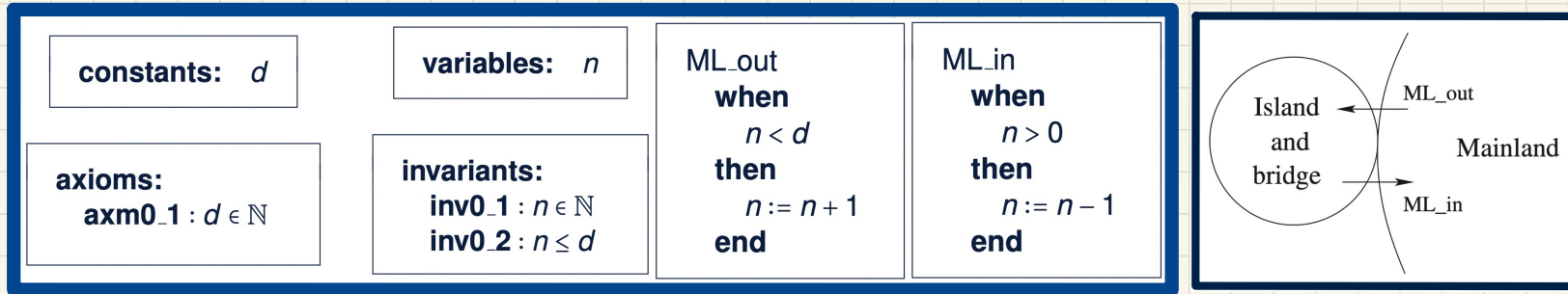


# Understanding the Failed Proof on **DLF**



**Unprovable** Sequent:  $\vdash d > 0$

# Discharging PO of **DLF**: Second Attempt

$$\begin{array}{l} d \in \mathbb{N} \\ n \in \mathbb{N} \\ n \leq d \\ \vdash \\ n < d \vee n > 0 \end{array}$$
 $\equiv$ 
$$\begin{array}{l} d \in \mathbb{N} \\ n \in \mathbb{N} \\ n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

**MON**

$$\begin{array}{l} n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

**OR\_L**

$$\begin{array}{l} n < d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

**OR\_R1**

$$\begin{array}{l} n < d \\ \vdash \\ n < d \end{array}$$

**HYP**

$$\begin{array}{l} n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

**EQ\_LR, MON**

$$\begin{array}{l} \vdash \\ d < d \vee d > 0 \end{array}$$

**OR\_R2**

$$\begin{array}{l} \vdash \\ d > 0 \end{array}$$

**?**

# Summary of the Initial Model: Provably Correct

**constants:**  $d$

**variables:**  $n$

**axioms:**

**axm0\_1** :  $d \in \mathbb{N}$

**axm0\_2** :  $d > 0$

**invariants:**

**inv0\_1** :  $n \in \mathbb{N}$

**inv0\_2** :  $n \leq d$

init  
**begin**  
     $n := 0$   
**end**

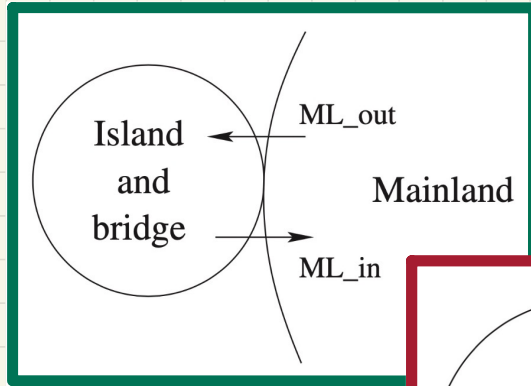
ML\_out  
**when**  
     $n < d$   
**then**  
     $n := n + 1$   
**end**

ML\_in  
**when**  
     $n > 0$   
**then**  
     $n := n - 1$   
**end**

**Correctness** Criteria:

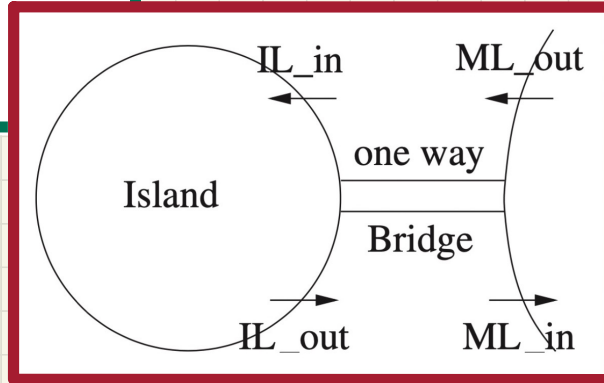
- + Invariant Establishment
- + Invariant Preservation
- + Deadlock Freedom

# Bridge Controller: **Abstraction** in the 1st Refinement



m0:

initial, most **abstract**



m1:

second, more **concrete**

REQ1

The system is controlling cars on a bridge connecting the mainland to an island.

REQ3

The bridge is one-way or the other, not both at the same time.

# Bridge Controller: State Space of the 1st Refinement

REQ1

The system is controlling cars on a bridge connecting the mainland to an island.

REQ3

The bridge is one-way or the other, not both at the same time.

## Dynamic Part of Model

**variables:**  $a, b, c$

**invariants:**

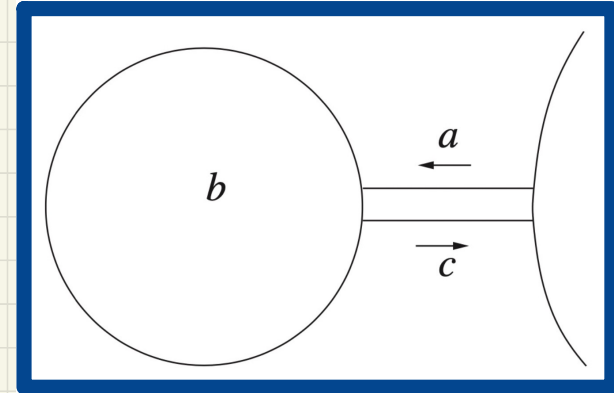
inv1\_1 :  $a \in \mathbb{N}$

inv1\_2 :  $b \in \mathbb{N}$

inv1\_3 :  $c \in \mathbb{N}$

inv1\_4 : ??

inv1\_5 : ??



## Static Part of Model

**constants:**  $d$

**axioms:**

axm0\_1 :  $d \in \mathbb{N}$

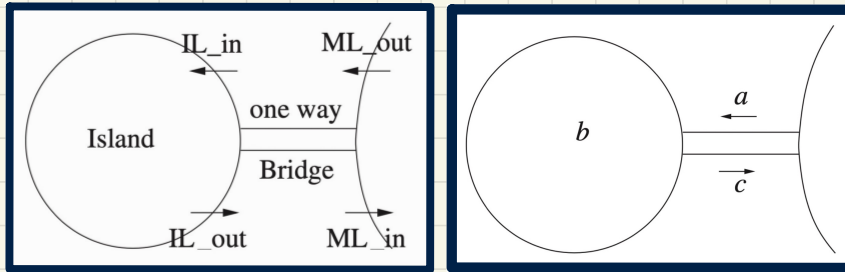
axm0\_2 :  $d > 0$

## Exercises

**inv1\_4**: linking abstract & concrete states

**inv1\_5**: bridge is one-way

# Bridge Controller: **Guards** of “old” Events 1st Refinement



**ML\_out**: A car exits mainland  
(getting on the **bridge**).

```
ML_out
when
  ??
then
  a := a + 1
end
```

**constants:**  $d$

**axioms:**

axm0\_1 :  $d \in \mathbb{N}$   
axm0\_2 :  $d > 0$

**ML\_in**: A car enters mainland  
(getting off the **bridge**).

```
ML_in
when
  ??
then
  c := c - 1
end
```

**variables:**  $a, b, c$

**invariants:**

inv1\_1 :  $a \in \mathbb{N}$   
inv1\_2 :  $b \in \mathbb{N}$   
inv1\_3 :  $c \in \mathbb{N}$   
inv1\_4 :  $a + b + c = n$   
inv1\_5 :  $a = 0 \vee c = 0$